FUZZY RULE-BASED REGRESSION FOR EXPLAINABLE AI

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EXPLAINABLE AI - MOTIVATION

- **Recent successes in AI [1]**
  - Primarily in Machine Learning (ML) and Deep Learning (DL)

[1] W. Ding et al., Explainability of Articial Intelligence Methods, Applications and Challenges: A Comprehensive Survey
[2] A. Adadi and M. Berrada, Peeking Inside the Black-Box: A Survey on Explainable Articial Intelligence (XAI)

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  - → Lack of trust
  - Critical domains: Healthcare, finance, cybersecurity, …

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Explain to **improve**	Explain to **discover**

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  - Critical domains: Healthcare, finance, cybersecurity, …
- Not only models explaining themselves but explain existing data
- Fuzzy rule-based regression is inherently explainable
  - Usable in many areas of XAI (see appendix for a simple taxonomy)

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SECTION 2

Introduction to Fuzzy Logic

FUNDAMENTALS OF FUZZY SETS

- **Published by Lotfi Zaddeh in 1965 [4]**
  - Traditionall set membership: $\mu_{trad}: \mathbb{R} \rightarrow \{0,1\}$

[4] L. Zadeh, Fuzzy Sets, Information and Control, vol. 8, no. 3, pp. 338-353
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    - $\mu_X\colon \mathbb{R} \to [0,1]$
    - $a \in \mathbb{R}$, fuzzy set $X, \mu_X(a) \in [0,1]$

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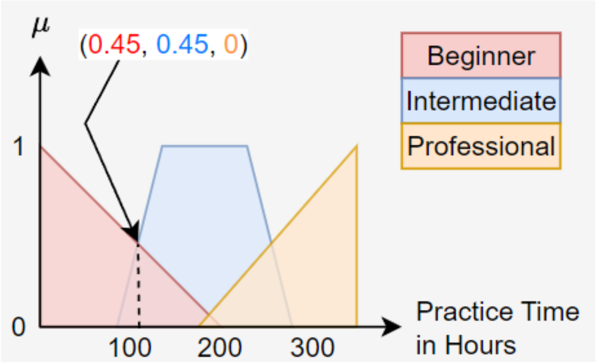

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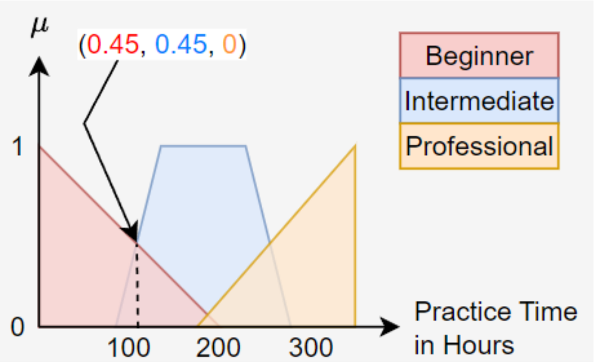

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  - Are bacteria anmials? [4]

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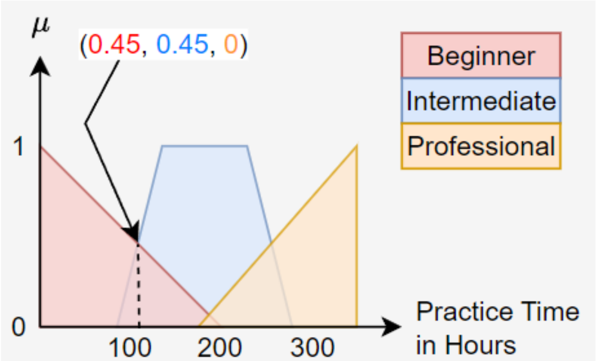

- **Vagueness is important for reasoning [5]**
  - Are bacteria anmials? [4]
- **Types of fuzzy sets**
  - Type 0: Crisp outputs of Takagi-Sugeno-Kang (TSK)-Systems (used in [6])
  - Type 1: Degree of membership (we will constrain ourselves to these)
  - Type 2: Model multiple sources of uncertainties in MFs

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FUNDAMENTALS OF FUZZY INFERENCE

- **Operations compute memberships:**
  - Let A, B be fuzzy sets, μ_A, μ_B be MFs, $a, b \in \mathbb{R}$.
    Furthermore, $x := \mu_A(a)$ and $y := \mu_B(b)$ are fuzzy variables
  - **AND** $\mu_{A \cap B}$:

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    - Other t-Norms with $t(x, y)$

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- **Often correlation instead of implication [9]**
  - 14+ types of fuzzy implications [10]

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FUNDAMENTALS OF FUZZY INFERENCE

Name	Symbol	Class	Function $\mathcal{J}(a, b)$	Axioms	Complement $c(a)$	Year
Early Zadeh	$\mathcal{J}_m$	QL	$\max[1-a, \min(a, b)]$	1, 2, 3, 4, 9	$1-a$	1973
Gaines-Rescher	$\mathcal{J}_s$		$\begin{cases} 1 & a \leq b \\ 0 & a > b \end{cases}$	1, 2, 3, 4, 5, 6, 7, 8		1969
Gödel	$\mathcal{J}_g$	R	$\begin{cases} 1 & a \leq b \\ b & a > b \end{cases}$	1, 2, 3, 4, 5, 6, 7		1976
Goguen	$\mathcal{J}_\Delta$	R	$\begin{cases} 1 & a \leq b \\ b/a & a > b \end{cases}$	1, 2, 3, 4, 5, 6, 7, 9		1969
Kleene-Dienes	$\mathcal{J}_b$	S, QL	$\max(1-a, b)$	1, 2, 3, 4, 6, 8, 9	$1-a$	1938, 1949
Lukasiewicz	$\mathcal{J}_a$	R, S	$\min(1, 1-a+b)$	1, 2, 3, 4, 5, 6, 7, 8, 9	$1-a$	1920
Pseudo-Lukasiewicz 1	$\mathcal{J}_\lambda$ $(\lambda > -1)$	R, S	$\min\left[1, \frac{1-a+(1+\lambda)b}{1+\lambda a}\right]$	1, 2, 3, 4, 5, 6, 7, 8, 9	$\frac{1-a}{1+\lambda a}$	1987
Pseudo-Lukasiewicz 2	$\mathcal{J}_w$ $(w > 0)$	R, S	$\min\left[1, (1-a^w+b^w)^{\frac{1}{w}}\right]$	1, 2, 3, 4, 5, 6, 7, 8, 9	$(1-a^w)^{\frac{1}{w}}$	1987
Reichenbach	$\mathcal{J}_r$	S	$1-a+ab$	1, 2, 3, 4, 6, 8, 9	$1-a$	1935
Willmott	$\mathcal{J}_{wl}$		$\min[\max(1-a, b), \max(a, 1-a), \max(b, 1-b)]$	4, 6, 8, 9	$1-a$	1980
Wu	$\mathcal{J}_{wu}$		$\begin{cases} 1 & a \leq b \\ \min(1-a, b) & a > b \end{cases}$	1, 2, 3, 5, 7, 8	$1-a$	1986
Yager	$\mathcal{J}_y$		$\begin{cases} 1 & a = b = 0 \\ b^a & \text{others} \end{cases}$	1, 2, 3, 4, 6		1980
Klir and Yuan 1	$\mathcal{J}_p$	QL	$1-a+a^2b$	2, 3, 4, 9	$1-a$	1994
Klir and Yuan 2	$\mathcal{J}_q$	QL	$\begin{cases} b & a = 1 \\ 1-a & a \neq 1, b \neq 1 \\ 1 & a \neq 1, b = 1 \end{cases}$	2, 4	$1-a$	1994

Table 11.1 from [10]

[10] D. K. Wedding, Fuzzy Sets and Fuzzy Logic: Theory and Applications: George J. Klir and Bo Yuan

FUNDAMENTALS OF FUZZY INFERENCE SYSTEMS (FIS)

- **Computational models built upon fuzzy logic**
  - = Fuzzy Inference Systems (FIS)
  - Popular variants: Mamdani and Takagi-Sugeno-Kang (TSK)

[8] K. Wang, Computational Intelligence in Agile Manufacturing Engineering: The 21st Century Competitive Strategy

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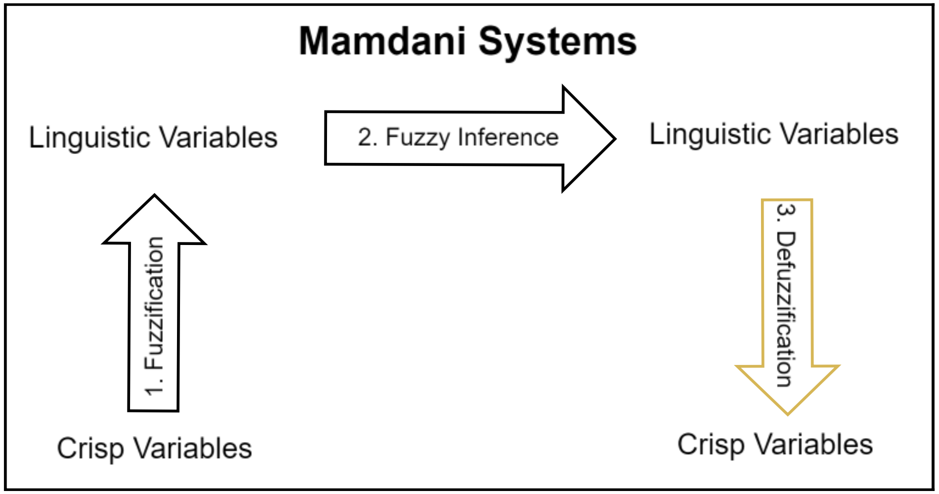

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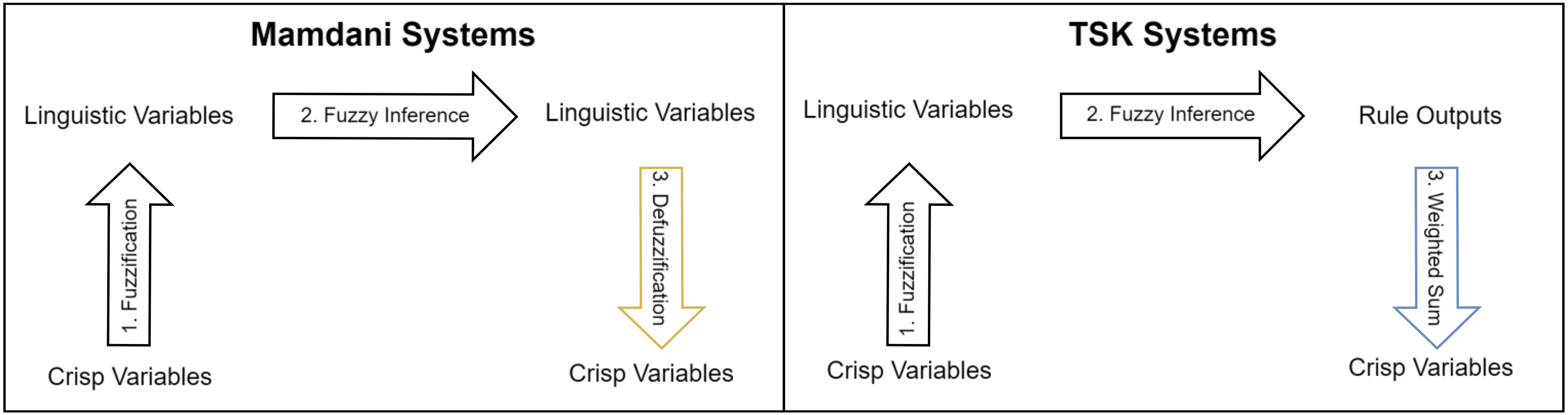

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EXAMPLE MAMDANI SYSTEM

Example: Competition success estimation for *Capture The Flag* competitions

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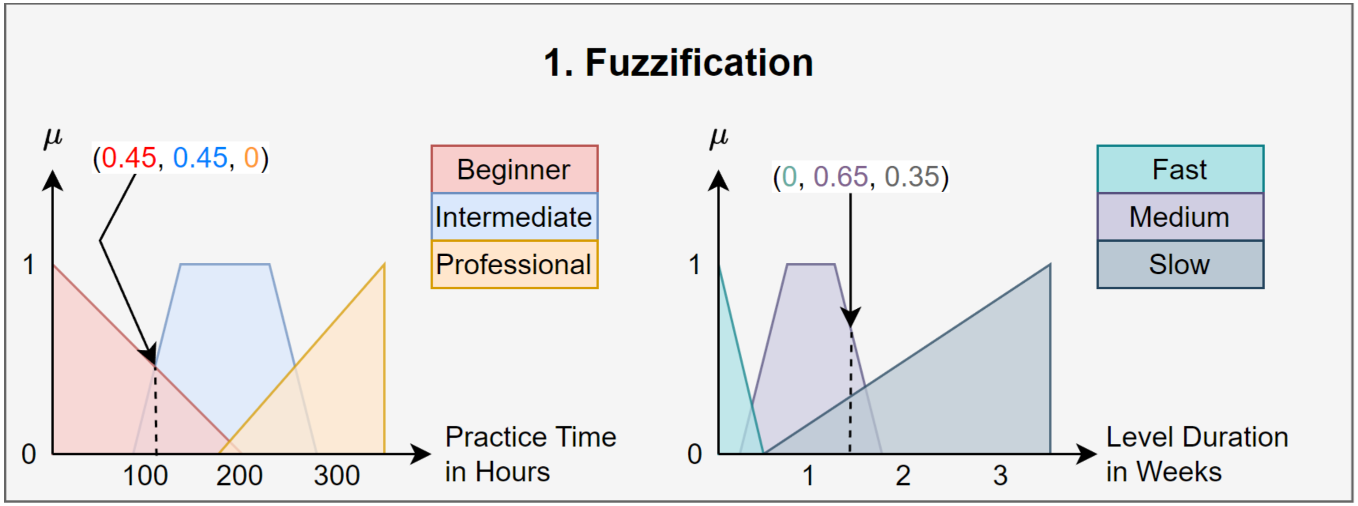
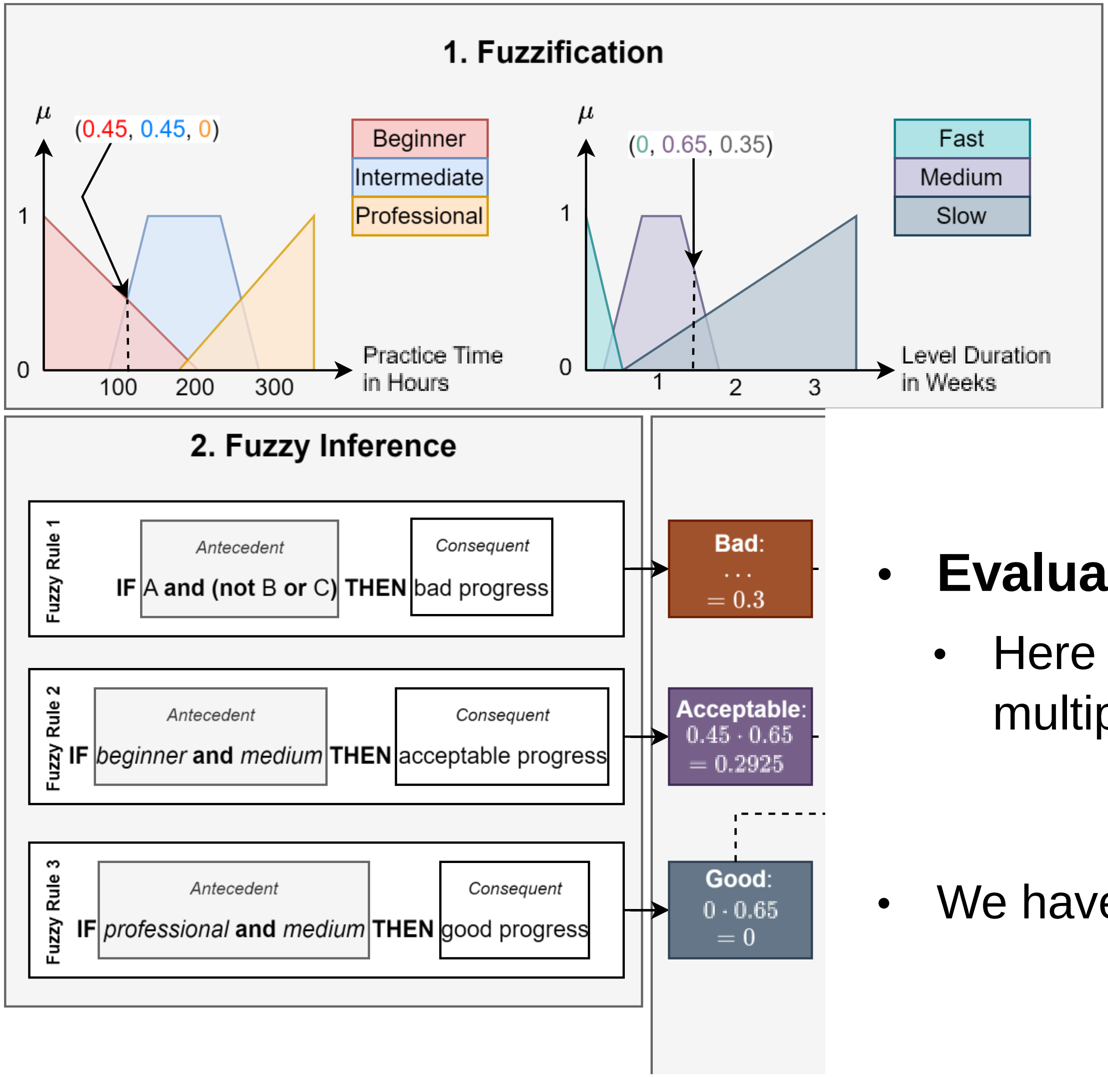


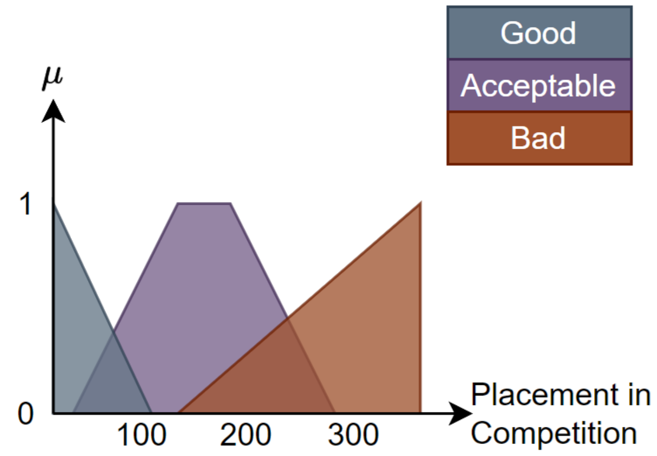
EXAMPLE MAMDANI SYSTEM: FUZZY INFERENCE



- **Evaluate Antecedents**
  - Here **AND** is modeled as multiplication
- We have natural language results!

MEMBERSHIP FUNCTION FOR DEFUZZIFICATION

- **Consequent will cap defuzzification**
  - $\min(\mu_X, c_X)$ for all sets X and consequent values c
  - Good = 0
  - Acceptable ≈ 0.3
  - Bad = 0.3



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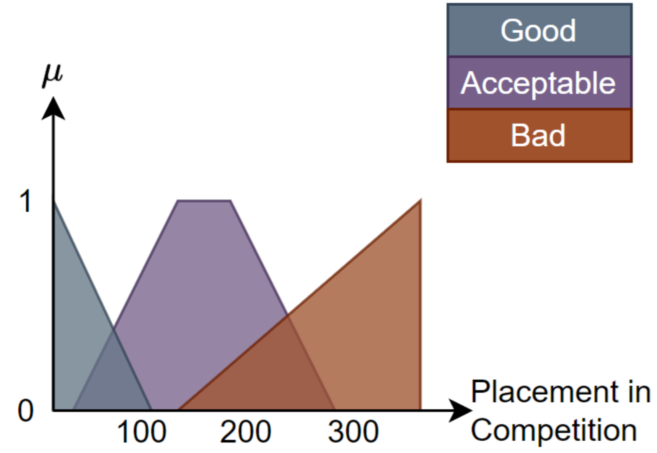
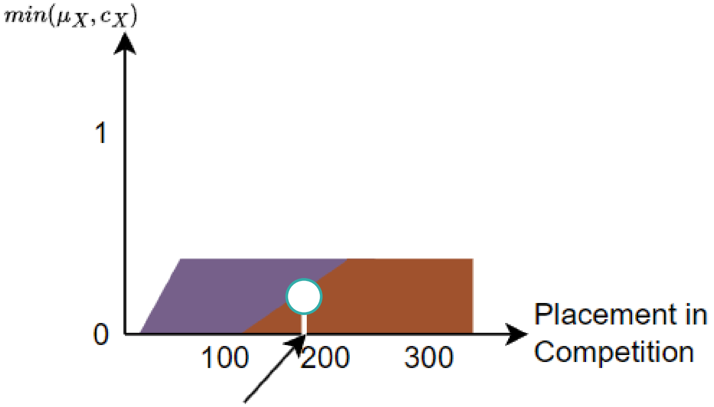
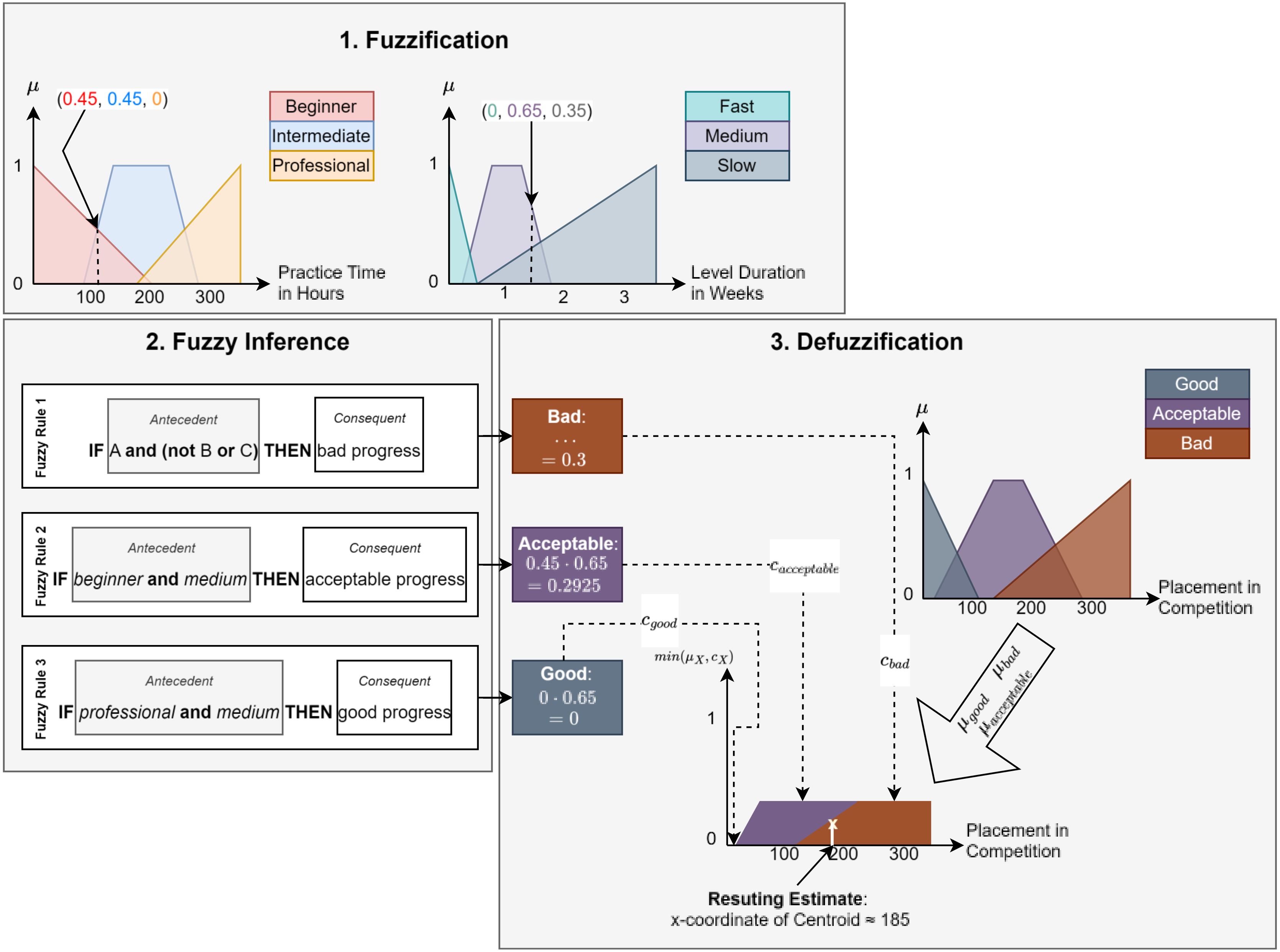


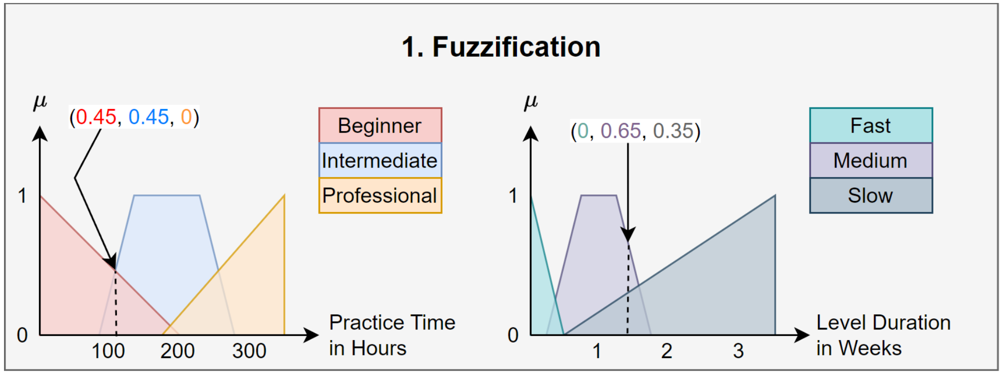
- **Domain-Value of MF-Centroid will be the output**
  - Only on the remaining shape




1. Fuzzification
μ
(0.45, 0.45, 0)
Beginner
Intermediate
Professional
1
0
100
200
300
Practice Time in Hours
μ
(0, 0.65, 0.35)
Fast
Medium
Slow
1
0
1
2
3
Level Duration in Weeks
2. Fuzzy Inference
Fuzzy Rule 1
Antecedent
Consequent
IF A and (not B or C) THEN bad progress
Fuzzy Rule 2
Antecedent
Consequent
IF beginner and medium THEN acceptable progress
Fuzzy Rule 3
Antecedent
Consequent
IF professional and medium THEN good progress
3. Defuzzification
Bad: ... = 0.3
Acceptable: 0.45 · 0.65 = 0.2925
Good: 0 · 0.65 = 0
Good
Acceptable
Bad
μ
1
0
100
200
300
Placement in Competition
c_acceptable
c_good
c_bad
min(μ_X, c_X)
μ_good μ_bad μ_acceptable
1
0
100
200
300
Placement in Competition
Resuting Estimate:
x-coordinate of Centroid ≈ 185

TAKAGI-SUGENO-KANG DEFUZZIFICATION

- **Inference outputs are “Type-0“ (=crisp) polynomials [11]**
  - Use antecedent values as weight for conclusions
  - Weighted sum as output



Example

h: Practice time in hours = 110
d: Level duration in weeks = 1.5

[11] T. Takagi and M. Sugeno, Fuzzy Identification of Systems and Its Applications to Modeling and Control

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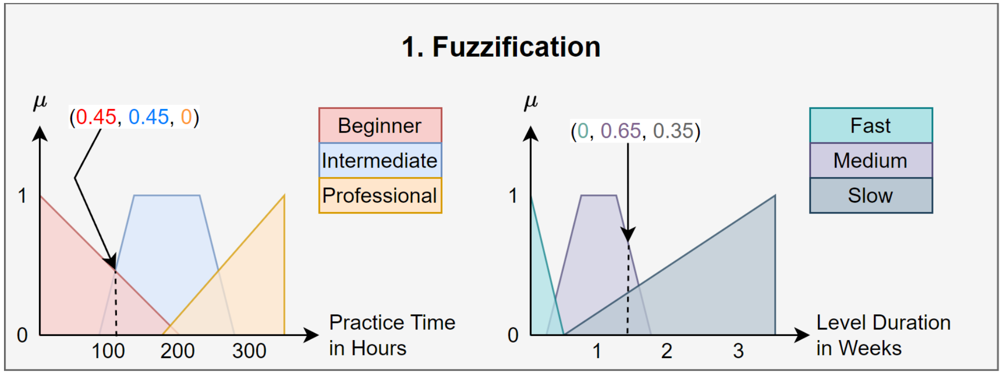

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IF beginner AND fast THEN placement = 150 – h + 20*d
IF professional OR medium THEN placement = 200-h
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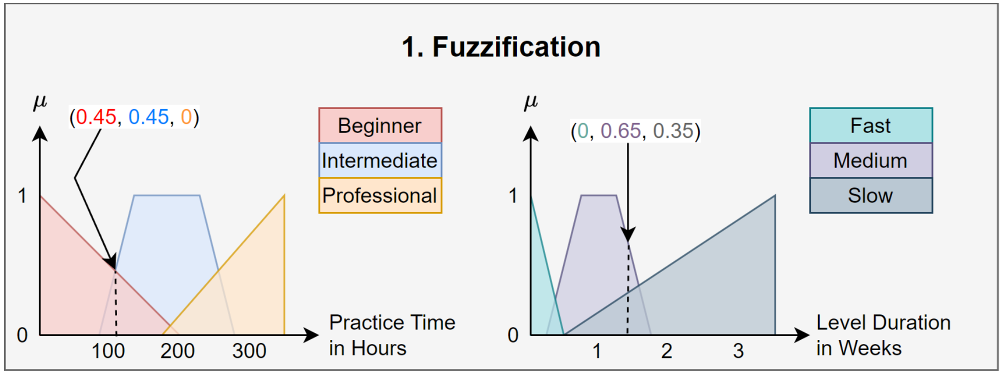

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We get Placement = $\frac{(0.45*0)*(150-110+20*1.5)+\max\{0,0.65\}*(200-h)}{(0.45*0)+\max\{0,0.65\}} = 90$

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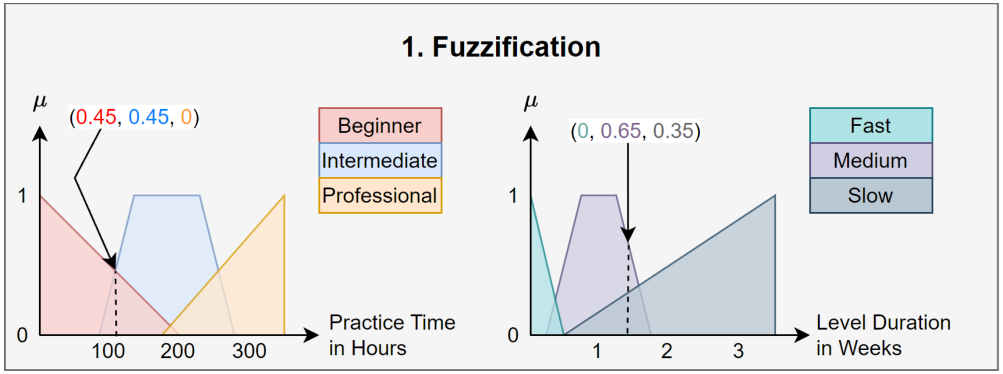

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VISUALIZING FUZZY INFERENCE SYSTEMS

Simple methods highlighted by [18]

[18] J. Cao, et al., Fuzzy Inference System With Interpretable Fuzzy Rules: Advancing Explainable Artifcial Intelligence for Disease Diagnosis

[12] https://github.com/tenaci-hand-grip/fuzzy-visualization, 05.04.2024

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Simple methods highlighted by [18]

Explicit Rule List

IF (A **AND** B) THEN C=2
IF (A **OR** D) THEN E=4, C=1
IF (G **OR** (**NOT** F)) THEN C=0

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Explicit Rule List with MF

	Antecedent parameter 1	Consequent parameters
Rule 1	$\mu = 0.61; \sigma^2 = 0.5$	$p^0 = 0.589$
Rule 2	$\mu = 0; \sigma^2 = 1$	$p^0 = 0.195$
...	...	...
Rule k	$\mu = 12; \sigma^2 = 4$	$p^3 = 0.613$

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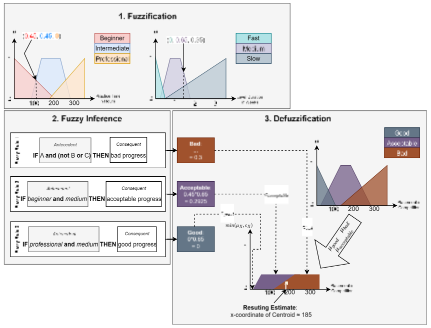
Showing Inference for One Sample



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[12] https://github.com/tenaci-hand-grip/fuzzy-visualization, 05.04.2024

VISUALIZING FUZZY INFERENCE SYSTEMS

Simple methods highlighted by [18]

Explicit Rule List

IF (A **AND** B) THEN C=2
IF (A **OR** D) THEN E=4, C=1
IF (G **OR** (**NOT** F)) THEN C=0

Explicit Rule List with MF

	Antecedent parameter 1	Consequent parameters
Rule 1	$\mu = 0.61; \sigma^2 = 0.5$	$p^0 = 0.589$
Rule 2	$\mu = 0; \sigma^2 = 1$	$p^0 = 0.195$
...	...	...
Rule k	$\mu = 12; \sigma^2 = 4$	$p^3 = 0.613$

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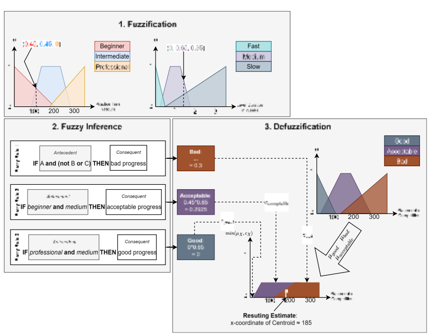

3D Surface-Plots

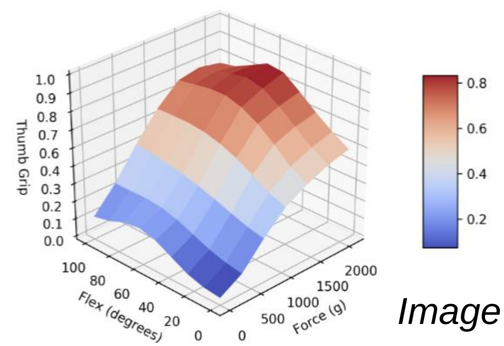

Image taken from [12]

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SECTION 3

Fuzzy Rule-Based Regression

EXAMPLE: ASSET RISK VALIDATION

- **Risk estimation of IT-Assets (BAKK)[1]**

1: Recommended to me by Univ.-Prof. Priv.-Doz. DDI Dr. Rass

EXAMPLE: ASSET RISK VALIDATION

- **Risk estimation of IT-Assets (BAKK)[1]**
  - You are the Chief Information Security Officer (CISO) of a big company
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  - Linear regression

    → Very high or low (negative coefficients) indicate important rules
  - Statistical testing for rule (ir-)relevance
  - **Explain the data generation process**
    - Deviations from best practices/agreed frameworks?
    - Conflicts or opposing actions?

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LINEAR REGRESSION OVER FIS

- **Goal: Find rules that explain the data**
- **Combine** $k \in \mathbb{N}$ **such FIS** $g_1, \ldots, g_k: \mathbb{R}^d \to \mathbb{R}$ **linearly** *similar to [13,14]*

[13] C. Cichy and S. Rass, A Fuzzy-Approximation-Approach to Explainable Information Quality Assessment
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  - Let $\boldsymbol{x_i} \in \mathbb{R}^d$ be the i-th input vector with d dimensions
  - Let $\alpha_0, \alpha_1, \ldots, \alpha_k \in \mathbb{R}$ be the **rule-weights** (parameters) we will tune later

$$f(x_i, \alpha_0, \alpha_1, \ldots, \alpha_k) = \alpha_0 + \alpha_1 g_1(\boldsymbol{x_i}) + \cdots + \alpha_k g_k(\boldsymbol{x_i})$$

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- **Minimize Ordinary Least Squares**
  - Let $y_i \in \mathbb{R}$ be the i-th output label (could also be defined multidimensional)

$$SSE = \sum_{i=1}^{n} \left(y_i - f(\boldsymbol{x_i}, \alpha_0, \alpha_1, \dots, \alpha_n)\right)^2$$

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  - Solution: Set first derivative to zero → normal equations [15]
    - No guarantee for global optimality → more examination/tries
    - Numerically more stable with QR-Decomposition

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REGRESSION OVER FUZZY INFERENCE SYSTEMS

- **Testing Significance Levels of Coefficients**
  - Can some g_i be dropped with same performance? [13]

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    - Statsitical test: Likelihood Ratio Test (let $\mathcal{L}$ denote likelihood)
    - H_0: Basis function g_i can be dropped
    - Choose:

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  - **Likelihood Ratio Test**: $\Lambda = 2(\log(\mathcal{L}_{full}) - \log(\mathcal{L}_{reduced}))$
    - Assume Gaussian error distribution: $\Lambda \sim \mathrm{X}^2$ [17]
      - Support: “Central Limit Theorem“ and “Principle of Maximum Entropy“
    - If Λ > $\mathrm{X}^2_{0.05,1}$(crit.) → reject H_0 → g_i is important, keep!

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  - Under Gaussian distribution assumptions for the data and Maximum Likelihood Estimation (MLE) for variance $\sigma^2 = \frac{SSE}{n}$ [16]:

$$\log \mathcal{L}(SSE, n) = -\frac{n}{2}\log(2\pi) - \frac{n}{2}\log\left(\frac{SSE}{n}\right) - \frac{n}{2}$$

n: number of samples
SSE: Sum of Squared Errors

  - n is equal so we only test the SSE of both models

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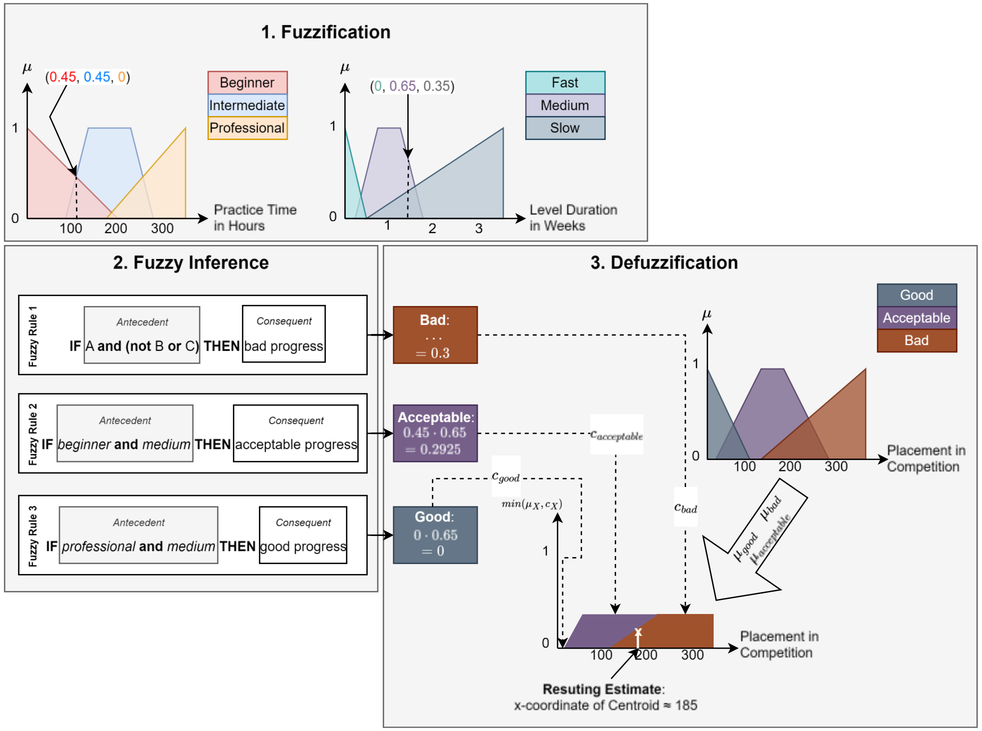
CONCLUSION



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  - Natural language
  - Many possible visualizations
- Determining rule importance by regressing over fuzzy systems

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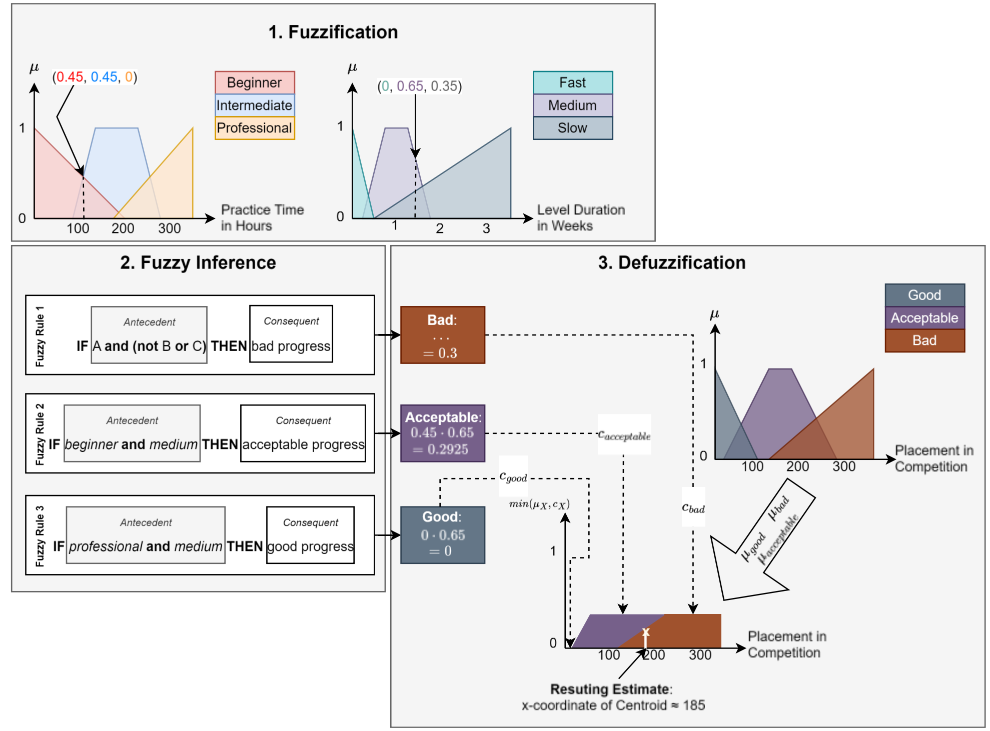

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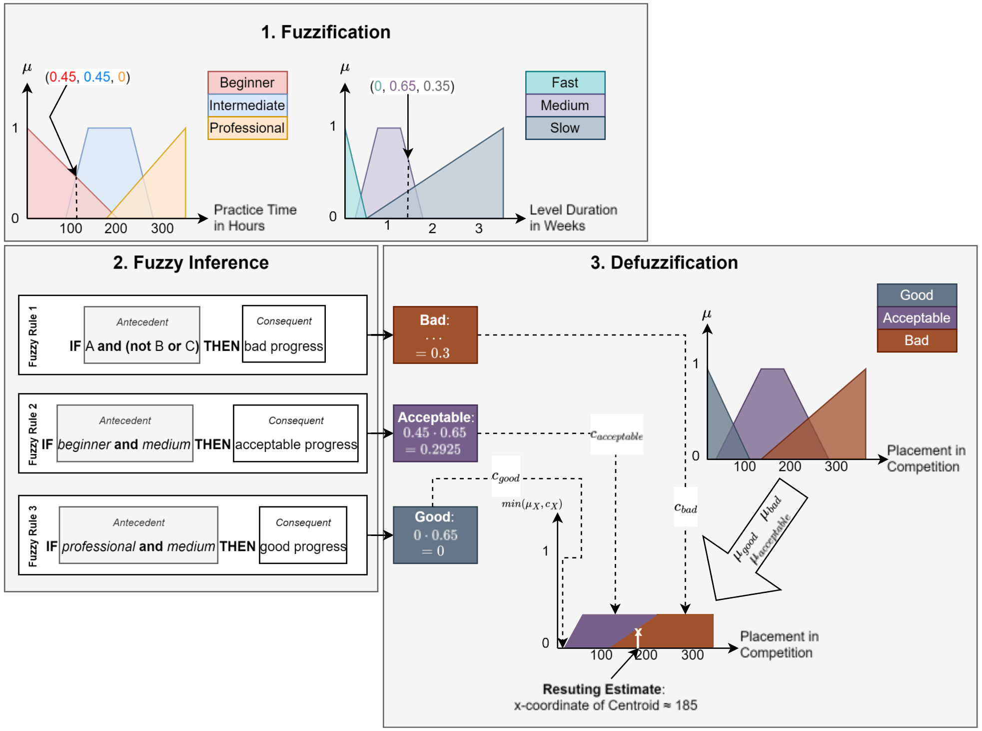

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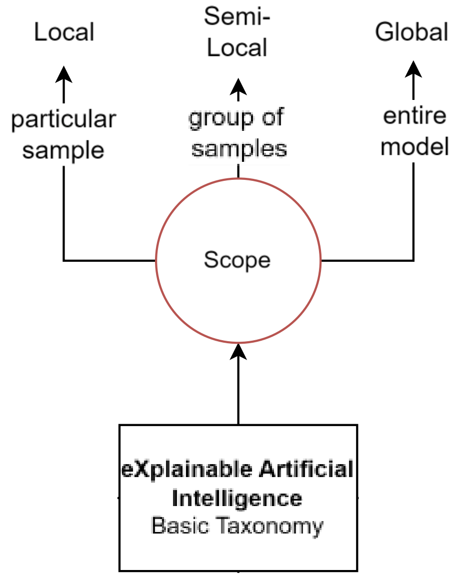
- Time for Questions!

APPENDIX

Martin Dallinger; 15.05.2024

JOHANNES KEPLER
UNIVERSITÄT LINZ
Altenberger Straße 69
4040 Linz, Österreich
www.jku.at

EXPLAINABLE AI – TAXONOMY[1,3]



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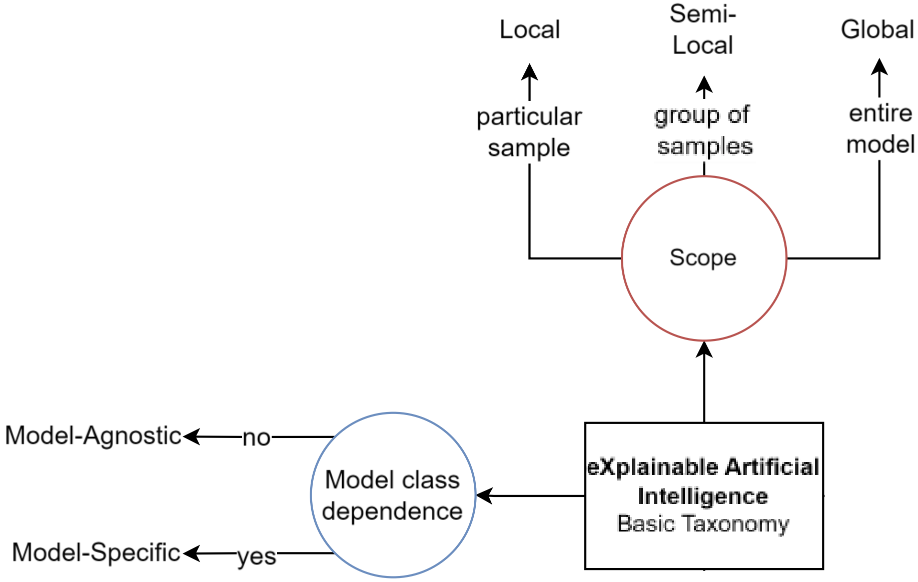

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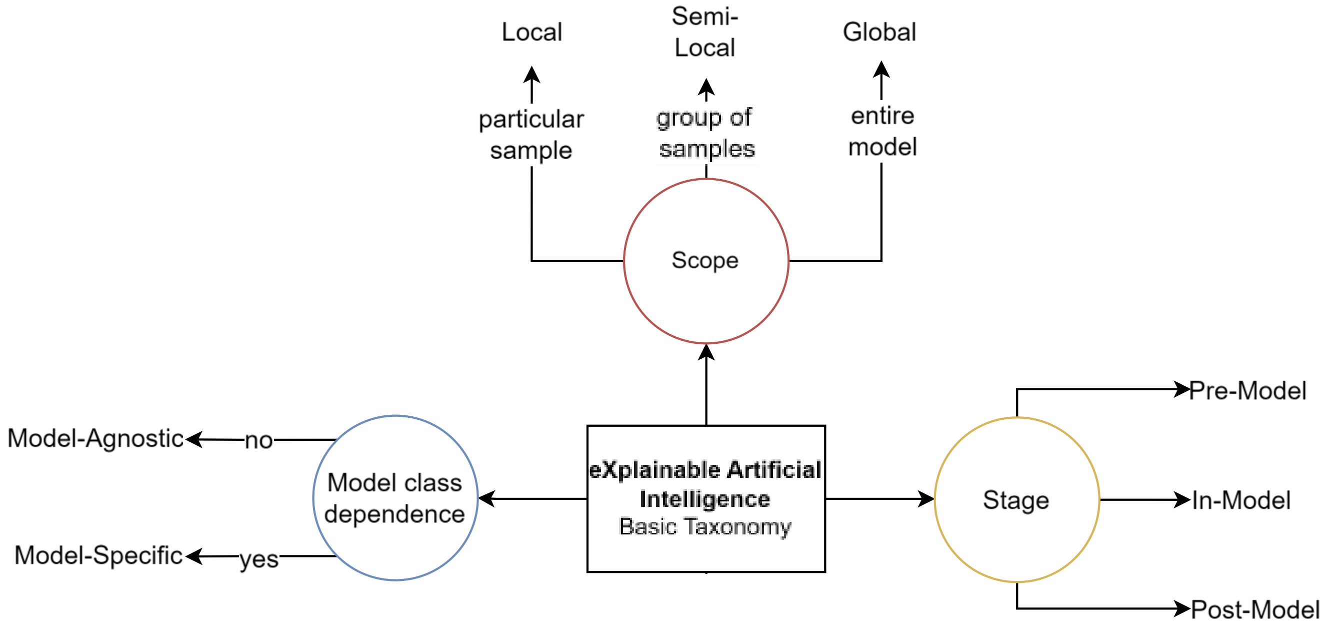

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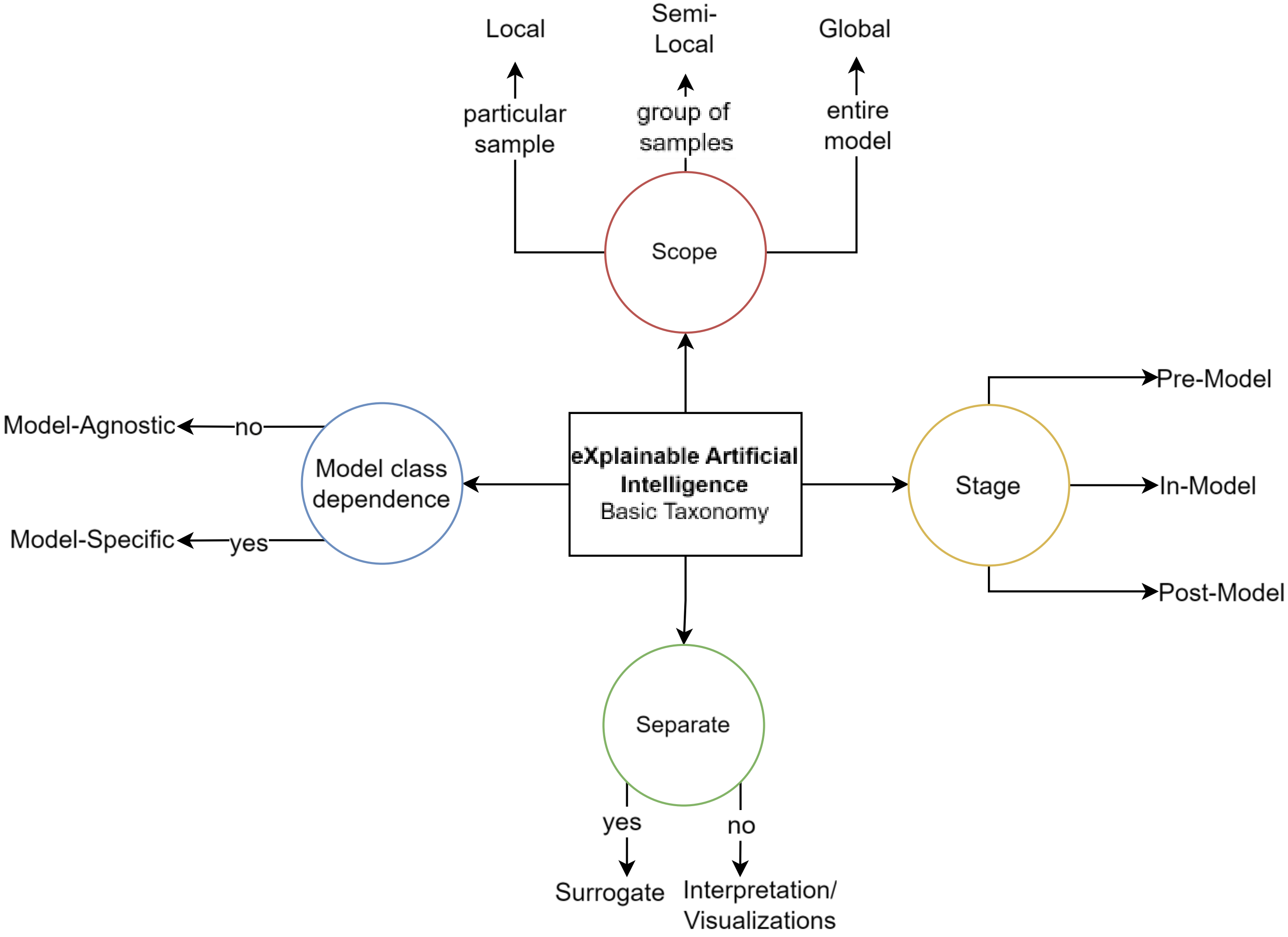

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FUNDAMENTALS OF FUZZY SETS CONT.

- **Type 1 fuzzy hedges [7]**
  - Linguistic operators
  - Enhance descriptive capabilities
  - “slightly, highly, very, more or less, much“

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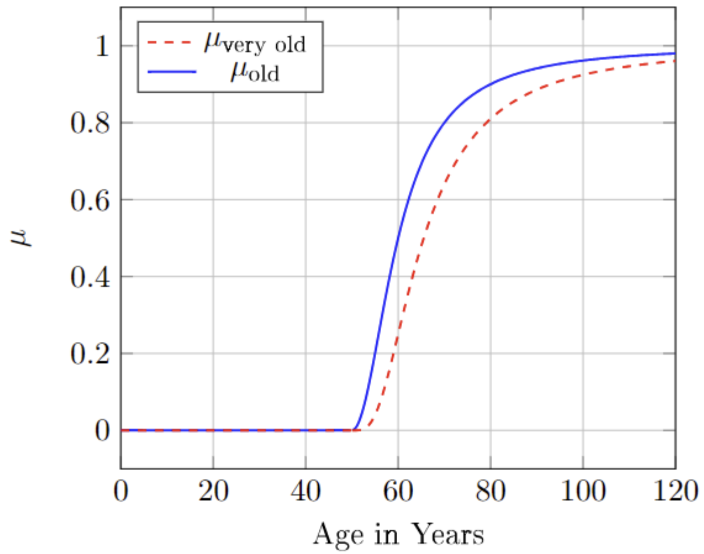

Image adapted from [7]

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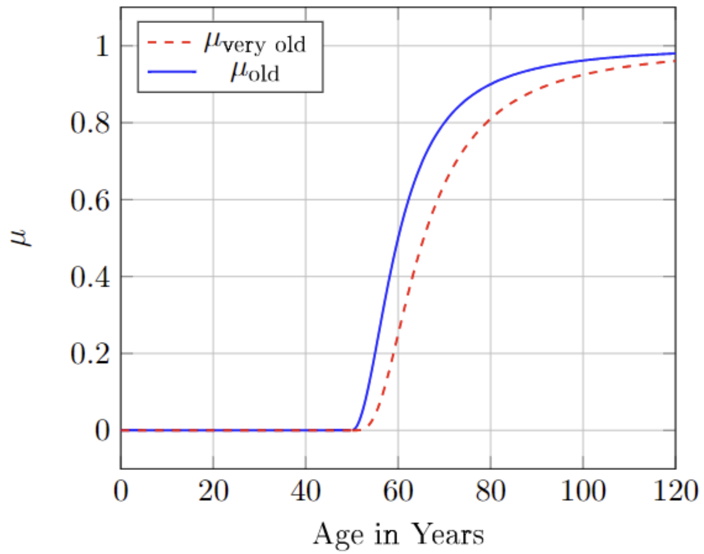
- **Type 2 fuzzy hedges [7]**
  - Concept-related beyond MFs (out of scope)
  - “safe“ vs. “essentially safe“ – basic criteria



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[12] Image: https://github.com/tenaci-hand-grip/fuzzy-visualization, 05.04.2024

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